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Stringy instantons and cascading quivers

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Abstract: D-brane instantons can perturb the quantum field theories on space-time filling D-branes by interesting operators. In some cases, these D-brane instantons are novel "stringy" effects (not interpretable directly as instanton effects in the low-energy quantum field theory), while in others the D-brane instantons can be directly interpreted as field theory effects. In this note, we describe a situation where both perspectives are available, by studying stringy instantons in quivers which arise at simple Calabi-Yau singularities. We show that a stringy instanton which wraps an unoccupied node of the quiver, and gives rise to a non-perturbative mass in the space-time field theory, can be reinterpreted as a conventional gauge theory effect by going up in an appropriate renormalization group cascade. Interestingly, in the cascade, the contribution of the stringy instanton does not come from gauge theory instantons but from strong coupling dynamics.

Keywords: Supersymmetric gauge theory, D-branes, Brane Dynamics in Gauge Theories, Nonperturbative Effects.

Contents

1. Introduction

Quantum effects which are non-perturbatively small in the coupling constant g may play an important role in many physical phenomena. For instance, they may be relevant to dynamical (super)symmetry breaking, or provide a natural mechanism to generate small Yukawa couplings or masses in the Lagrangian. In string theory, instantons which generate such effects can often be geometrized as D-branes. Investigation of both novel stringy effects (not obviously interpretable as field theory instanton effects) [1–3] and more conventional field theory instanton effects [4, 3, 5] involving Euclidean D-branes has recently been initiated by several groups (building in part on important earlier work of Ganor [6] and Witten [7]). Further explorations and applications of these instantons appear in [8–20].

A rich set of theories where such effects may be computable and important is provided by D-branes at Calabi-Yau singularities. Our goal in this paper will be to use such a system to demonstrate a D-brane instanton effect in two different and complementary ways. Branes at singularities give rise rather generally to quiver gauge theories [21]. If some nodes of the quiver are unoccupied by space-filling branes, one may still construct interesting instantons by wrapping Euclidean branes over the corresponding cycles in the geometry [3]. These can give rise to stringy perturbations of the low-energy effective theory, which are not directly interpreted as instanton effects in the quiver gauge theory.

On the other hand, many quivers corresponding to four dimensional $\mathcal{N} = 1$ supersymmetric gauge theories exhibit renormalization group (RG) cascades [22]. These include D-branes at conifolds and their generalizations (but not orbifold singularities, which have free worldsheet descriptions). The cascades describe brane systems where all nodes of the quiver are occupied at high energies in the field theory, while the low energy physics may be described by a brane configuration with some unoccupied nodes.

In orbifolds, one can directly use worldsheet techniques to systematically derive the string instanton contributions. While this is not possible for conifolds and their generalizations, in such systems, one instead has the intriguing possibility of computing the D-brane instanton effect generated by a brane wrapping an unoccupied quiver node, in two different ways:

(i) One can do the path integral over D-instanton collective coordinates in the quiver with occupation numbers describing the end of the cascade. This requires use of sophisticated mathematical technology to infer the action on the D-instanton [23].

(ii) One can try to derive the *same* effect by analyzing the gauge theory at higher steps in the cascade, where the relevant node is occupied by space-filling branes. In this case one should be able to reproduce the desired effect by using standard techniques in $\mathcal{N} = 1$ supersymmetric gauge theory.

The agreement between methods (i) and (ii) that we find (in a particular simple system that we can analyze in detail), can be viewed as a consistency check on the presence and form of the novel stringy effects. Somewhat surprisingly, in the second method we find that the effect does not come from field theory instantons but from strong coupling dynamics.

The agreement between the two methods is intuitively expected for the following reason. The gauge theory at higher steps in the cascade in method (ii) can be UV completed by embedding an appropriate brane configuration in a non-compact, singular Calabi-Yau manifold.[1] We can also do this directly with the partially occupied quiver of method (i). The superpotential of the low-energy field theory is not expected to depend on the number of cascade steps K. As we lower K, eventually the gauge theory effect that we compute by method (ii), becomes a stringy effect that we compute by method (i).

The organization of this paper is as follows. In section 2, we introduce the singularity and the IR brane configuration that we will study. This configuration was previously explored in [25, 12]. In section 3, we review the expected stringy instanton effect. In section 4, we describe an RG cascade which ends with this brane configuration. In section 5, we show that careful analysis of the field theory dynamics along the cascade reproduces the expected result of section 3.

Although our analysis is limited to one illustrative case, we expect that similar results could be obtained for instanton computations in quivers characterizing more general Calabi-Yau singularities.

2. The quiver of branes at orbifolds of the conifold

Branes at singularities provide interesting gauge theories which exhibit reduced supersymmetry and intricate (non-conformal) IR dynamics, and which in many cases admit dual gravity descriptions. The simplest cases involve orbifold [21, 26, 27] and conifold [28] singularities.

[1] The cascade does not necessarily need to be completed in this manner. The cascade with an infinite number of steps can be defined by the holographic renormalization of its gravity dual [24].

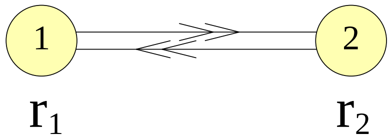

Figure 1: Quiver diagram for the conifold singularity. Each node is an $SU(r_i)$ gauge group, and each arrow is a bifundamental field. The difference in ranks counts the number of fractional D5-branes.

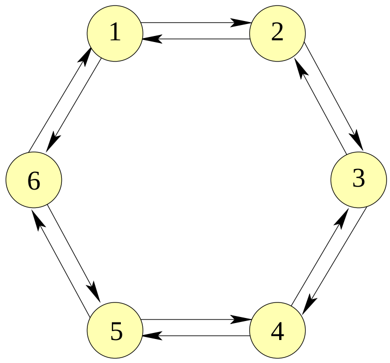

Figure 2: The quiver diagram of the orbifolded model for $n = 3$.

The conifold can be described by the equation

$$xy = zw \tag{2.1}$$

in $\mathbb{C}^4$. The quiver which captures the field theory on D3-branes and fractional D5-branes at this singularity, in type IIB string theory, is shown in figure 1.

The theory has a superpotential of the schematic form

$$W = h \text{ tr} \left(\epsilon_{ij}\epsilon_{kl}A^i B^k A^j B^l \right) \tag{2.2}$$

where the A's and B's represent the bifundamentals, and i, j (k, l) are global SU(2) indices for the flavor symmetry rotating the A's (B's). If one chooses $r_1 = N$, $r_2 = N + M$ with $N \gg M$, this theory enjoys an RG cascade described in [22].

A simple generalization of this singularity can be obtained by taking a $\mathbb{Z}_n$ quotient. One class of such orbifolds [29] is described by the equation

$$(xy)^n = zw \tag{2.3}$$

in $\mathbb{C}^4$. Branes at this singularity are governed by the quiver field theory with $2n$ nodes and bifundamentals of both chiralities going between adjacent nodes, as in figure 2. This theory enjoys a quartic superpotential of the form

$$W = \sum_i (-1)^i h Q^{(i)} Q^{(i+1)} \widetilde{Q}^{(i+1)} \widetilde{Q}^{(i)} \tag{2.4}$$

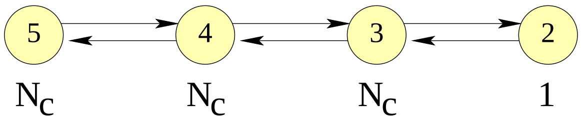

Figure 3: The gauge theory of interest, which can be engineered in any of the geometries above with $n \geq 3$.

where $Q^{(i)}$ corresponds to the arrow pointing between node i and node $i+1$ and so forth, and the contraction on gauge indices is the obvious one. If one is interested in a cascading solution with all ranks deviating from some large N by finite amounts in the UV, one can think of the matter fields Q as having dimension 3/4, and the parameter h as being (to leading order in $1/N$) dimensionless. In the IR, where the solution departs significantly from its approximation by the CFT which exists when all ranks are equal, it is more appropriate to think of h as being the inverse of some mass scale which is larger than any of the gauge theory dynamical scales $\Lambda^{(i)}$.

The class of quivers depicted in figure 2 was studied in [25, 12], as a simple home for metastable supersymmetry breaking vacua in string theory. The subquiver which was relevant there, and which should arise in the IR limit of any proposed UV completion, is shown in figure 3. For appropriate choices of the dynamical scales $\Lambda^{(i)}$ associated with nodes 3, 4 and 5, this theory was argued to give rise (at low-energy) to a SUSY QCD theory with $N_f = N_c + 1$ slightly massive flavors, and hence to admit metastable vacua analogous to those described in [30]. (A similar construction was described in [31]).

The only relevant point for us is that one of these non-vanishing masses arises from a stringy instanton, wrapping the (unoccupied) node to the right of the $\mathrm{U}(1)$ factor in the full quiver diagram. This instanton was argued, in suitable circumstances, to give rise to a mass term

$$W = \cdots + mQ^{(2)}\widetilde{Q}^{(2)} \tag{2.5}$$

for the $N_c + 1$'st quark flavor of the gauge group at node 3.

3. The stringy instanton

First, we should specify the circumstances in which the instanton is believed to contribute. In the absence of the space-filling branes, the instanton would break half of the supersymmetry in an $\mathcal{N} = 2$ supersymmetric Calabi-Yau compactification. Hence, one could obtain four fermion zero modes by acting on the instanton solution with the broken supercharges. These arise in the sector of open strings stretching from the instanton to itself.

However, to generate a contribution to the space-time superpotential, there should only be two fermion zero modes (that aren't soaked up in the integral over instanton collective coordinates). Although the other branes present in the quiver gauge theory do break the space-time supersymmetry to $\mathcal{N} = 1$, they do not apparently lift the extra two fermion zero modes. Therefore, to obtain a contribution to the superpotential, one should either:

a) Consider a slightly modified configuration, where the instanton wraps a node that also intersects an orientifold plane [10, 13 – 15]. The orientifold projection can eliminate

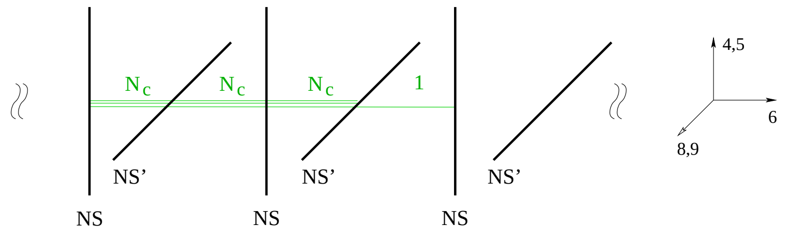

Figure 4: Type IIA T-dual description of the type IIB quiver in figure 3.

precisely half of the fermion zero modes in the relevant open string sector, leaving the needed two.

b) Consider a full compactification with background fluxes (and perhaps other ingredients) in the vicinity of the instanton. On general grounds, one would expect that in such cases, the brane could locally detect that the background preserves only $\mathcal{N} = 1$ supersymmetry, and would have only two zero modes generated by acting with broken supercharges. It is important to study the precise circumstances in which this happens, of course. Results in this direction, for instantons which do not intersect space-filling D-branes, are implicit in [7] (where the geometry of F-theory reduces the number of zero modes on certain D3-instantons to the required two), and are generalized to models with flux in [32 – 37].

We shall proceed with option (a). In fact, a class of orientifold models which leave the gauge theory on the space-filling branes unmodified, while allowing the instanton to intersect an O-plane, was already described in [12, 18].

It is easier to describe the relevant geometries in the T-dual type IIA picture. Recall that e.g. the quiver in figure 3 can be T-dualized to a type IIA brane configuration. The occupation numbers of the nodes map to numbers of D4 branes stretching on a circle in the x_6 direction between NS 5 branes stretched in the 012345 directions and NS 5' branes stretched in the 012389 directions — the NS and NS' branes alternate as one goes around the circle. Our configuration is shown in figure 4.

To make a quiver gauge theory which keeps the physics at our occupied nodes unchanged, while allowing the D-brane instanton at node 1 to acquire the required zero mode content, we introduce orientifolds as in figure 5. This configuration can be obtained by orientifolding the $n = 5$ model in an obvious way. The O6$^-$-planes extend along the 01237 directions, and lie at a 45 degree angle with respect to the 45 and 89 planes.

In this geometry (or the generalization including any number of additional SU(N_i) nodes between the O-planes), the D-instanton wrapping the node to the right of the SU(1) node, i.e. wrapping node 1, has SO(1) worldvolume gauge group (while space-filling branes occupying the same node would have symplectic gauge groups). This *SO* projection lifts the extra two fermion zero modes, so this D-instanton can potentially contribute to the space-time superpotential. It has in addition collective coordinates ("Ganor strings" [6]) which stretch to node 2, as in the extended quiver diagram shown in figure 6.

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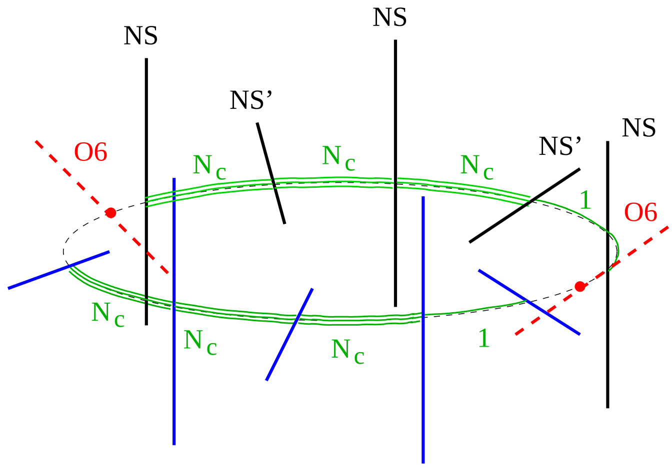

Figure 5: An orientifold which preserves our gauge theory and contains the required instantons.

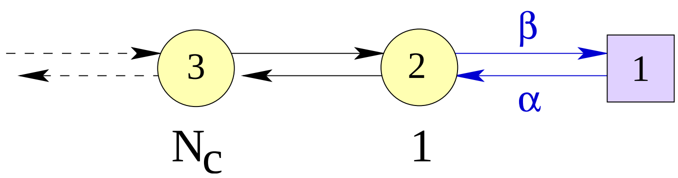

Figure 6: The extended quiver diagram, including a node for the Euclidean D-brane.

The instanton action is expected to contain a quartic coupling

$$S = \ldots + \alpha Q^{(2)} \widetilde{Q}^{(2)} \beta \, . \tag{3.1}$$

After integrating out the α, β collective coordinates, one will be left in the space-time theory with a mass term (2.5), with m suppressed by the exponential of the area A of node 1. Note that this effect only depends on nodes 1, 2 and 3, and should arise in any theory containing these nodes (as depicted in figure 6).

In the next sections we will derive the same superpotential (2.5) in an alternative way, by embedding the gauge theory in a renormalization group cascade. In this picture, the effect of the stringy instanton is reproduced by a gauge theory computation, and the exponential suppression of m due to the instanton action becomes exponential suppression by the dynamical scales of asymptotically-free gauge group factors (which confine and disappear at low energies).

4. The cascade and its orientifold

4.1 The $\mathrm{U}(N_i)$ cascade

In this subsection we briefly review the cascade in the quiver theories of section 2. At some energy scale, these theories can be described as a $\prod_{i=1}^{2n} \mathrm{SU}(N_i)$ gauge theory, with chiral multiplets $Q^{(i)}$ and $\widetilde{Q}^{(i)}$ $(i = 1, \cdots, 2n)$ in the $(\mathbf{N_i}, \overline{\mathbf{N_{i+1}}})$ and $(\overline{\mathbf{N_i}}, \mathbf{N_{i+1}})$ representations, respectively ($i+1$ is defined modulo $2n$). The quiver diagram for this theory is depicted in figure 2; it is a generalization of the original cascade of Klebanov and Strassler [22] which arises for $n=1$. The theory has an effective superpotential (in an arbitrary normalization of the fields) of the form

$$W=\sum_{i=1}^{2n}(-1)^i Q^{(i)}Q^{(i+1)}\widetilde{Q}^{(i+1)}\widetilde{Q}^{(i)}, \tag{4.1}$$

with the obvious contractions of indices. Throughout this section we will ignore numerical factors which will not be important for our considerations.

In this subsection we will analyze a generic high-energy step of the cascade, assuming that all the N_i's are large and comparable. Each gauge group in this theory has some strong coupling scale $\Lambda^{(i)}$. The general analysis of this theory is very complicated, but the analysis becomes simple when there are large ratios between all the scales $\Lambda^{(i)}$; then we can analyze the dynamics of each gauge theory separately as we go down in energy, ignoring the dynamics of the other gauge groups. A particularly simple ordering is

$$\Lambda^{(1)} \gg \Lambda^{(3)} \gg \cdots \gg \Lambda^{(2n-1)} \gg \Lambda^{(2)} \gg \cdots \gg \Lambda^{(2n)}. \tag{4.2}$$

In the order (4.2) the $\mathrm{SU}(N_1)$ theory, which has $N_f = N_2 + N_{2n}$ becomes strongly coupled first. Ignoring all other interactions (including the superpotential interactions which are assumed to be small at the scale $\Lambda^{(1)}$), the infrared dynamics of this theory is the same as that of its Seiberg dual [38], so we can use the variables of the dual theory instead of our original variables (a detailed description of the justification for this may be found in [39]). The dual is an $\mathrm{SU}(N_f - N_1) = \mathrm{SU}(N_2 + N_{2n} - N_1) = \mathrm{SU}(\widehat{N}_1)$ gauge theory, whose degrees of freedom are quarks $q^{(1)}$, $\widetilde{q}^{(1)}$, $q^{(2n)}$ and $\widetilde{q}^{(2n)}$ in the $(\widehat{\mathbf{N}}_{\mathbf{1}}, \mathbf{N_2})$, $(\overline{\widehat{\mathbf{N}}_{\mathbf{1}}}, \overline{\mathbf{N_2}})$, $(\overline{\mathbf{N_{2n}}}, \overline{\widehat{\mathbf{N}}_{\mathbf{1}}})$, $(\mathbf{N_{2n}}, \widehat{\mathbf{N}}_{\mathbf{1}})$ representations, respectively, and mesons $M^{(2n,2n)}$ (in the adjoint+singlet representation of $\mathrm{SU}(N_{2n})$), $M^{(2,2)}$ (in the adjoint+singlet representation of $\mathrm{SU}(N_2)$), $M^{(2n,2)}$ (in the $(\mathbf{N_{2n}}, \overline{\mathbf{N_2}})$ representation) and $M^{(2,2n)}$ (in the $(\overline{\mathbf{N_{2n}}}, \mathbf{N_2})$ representation). The superpotential of this theory, including the relevant terms from (4.1) (translated into the new variables), takes the form (with obvious contractions)

$$\begin{aligned} W \;=\; & M^{(2,2)}q^{(1)}\widetilde{q}^{(1)} + M^{(2n,2n)}q^{(2n)}\widetilde{q}^{(2n)} + M^{(2,2n)}\widetilde{q}^{(2n)}\widetilde{q}^{(1)} + M^{(2n,2)}q^{(1)}q^{(2n)} \\ & + M^{(2,2n)}M^{(2n,2)} - M^{(2,2)}Q^{(2)}\widetilde{Q}^{(2)} - M^{(2n,2n)}Q^{(2n-1)}\widetilde{Q}^{(2n-1)}. \end{aligned} \tag{4.3}$$

The mesons $M^{(2n,2)}$ and $M^{(2,2n)}$ are massive so they can be integrated out as we go to lower scales; this leaves a superpotential of the form

$$\begin{aligned} W \;=\; & M^{(2,2)}q^{(1)}\widetilde{q}^{(1)} + M^{(2n,2n)}q^{(2n)}\widetilde{q}^{(2n)} - q^{(2n)}q^{(1)}\widetilde{q}^{(1)}\widetilde{q}^{(2n)} \\ & - M^{(2,2)}Q^{(2)}\widetilde{Q}^{(2)} - M^{(2n,2n)}Q^{(2n-1)}\widetilde{Q}^{(2n-1)}. \end{aligned} \tag{4.4}$$

The theory at lower scales has been modified in three important ways (in addition to small changes in the charge assignments); instead of an $\mathrm{SU}(N_1)$ gauge theory we have an $\mathrm{SU}(N_2 + N_{2n} - N_1)$ theory, we have two additional sets of adjoint+singlet fields for the $\mathrm{SU}(N_2)$ and $\mathrm{SU}(N_{2n})$ nodes, and these fields have trilinear superpotential couplings to all

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the quarks surrounding them, which replace the two quartic couplings centered on these two nodes.

We can now perform a similar analysis for the $\mathrm{SU}(N_3)$ node, which will become dualized to an $\mathrm{SU}(N_2+N_4-N_3)$ theory. Most of the analysis is the same, except that one of the trilinear couplings in (4.4) now becomes a mass term for $M^{(2,2)}$ and for the field with the same quantum numbers arising as a meson in the $\mathrm{SU}(N_3)$ theory. Thus, these fields can also be integrated out, and after this step we are left with no adjoint for the $\mathrm{SU}(N_2)$ theory, and its quartic coupling is reinstated (by integrating out the massive mesons), with an opposite sign compared to the original quartic coupling in (4.1). Thus, after this step we get a theory in which the $\mathrm{SU}(N_{2n})$ and $\mathrm{SU}(N_4)$ nodes are modified (by adding adjoints and their associated couplings) and the other nodes are not. If we now continue to perform a similar analysis for the $\mathrm{SU}(N_5)$, $\mathrm{SU}(N_7)$, $\cdots$, $\mathrm{SU}(N_{2n-1})$ theories, we eventually end up with a theory of the same form as we started with; in the last step we have two sets of adjoint fields becoming massive. The only differences (except for the overall sign of the superpotential) are that all fundamentals of the odd nodes have become anti-fundamentals (and vice versa), and the ranks of the odd nodes have been modified to $\mathrm{SU}(N_i) \to \mathrm{SU}(N_{i-1}+N_{i+1}-N_i)$. The total rank of all groups together has been reduced by

$$N_{\text{cascade}} = 2\sum_{i=1}^{n}(N_{2i-1}-N_{2i}); \tag{4.5}$$

this must be positive in order to have a cascade and to be consistent with (4.2).

Obviously, we can now perform a similar analysis for all the even nodes. After the additional n steps we again go back to a theory of the same form, with the total rank of all groups again reduced by the same amount N_{cascade}. The field representations and the overall sign of the superpotential are now the same as in the original theory we started from. We can continue cascading down in energy, going back to the same theory (with reduced ranks) after every $2n$ steps. Eventually, the ranks become small enough so that the group that becomes strongly coupled goes outside the range $N_c+2 \le N_f < 3N_c$ where we can use Seiberg duality as above. One then has to check in detail what happens next; in some cases one obtains confinement in the IR theory.

Finally, let us discuss the global symmetries. There are $(2n)$ obvious vector-like $\mathrm{U}(1)$ symmetries, acting on each pair $Q^{(i)}$, $\widetilde{Q}^{(i)}$. In addition, we have in the classical theory a $\mathrm{U}(1)_R$ symmetry (with $(Q^{(i)},\widetilde{Q}^{(i)})$ both having charge 1/2) and a single axial $\mathrm{U}(1)_A$ symmetry which is preserved by the superpotential (4.1), under which $(Q^{(i)},\widetilde{Q^{(i)}})$ both have charge $(-1)^i$. For generic values of the N_i which lead to a cascade, both of these symmetries are anomalous. Note that these two $\mathrm{U}(1)$ symmetries do not remain fixed along the cascade; if we make some charge assignment at some energy scale, we get a different charge assignment after going through a cascade step (with explicit factors of Λ's in the superpotential to fix its charge). However, we can always rescale the bi-fundamentals at each step of the cascade by some factors of Λ's to go back to the simpler charge assignments described above.

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4.2 The orientifolded cascade

We will now discuss a particular orientifold of this cascade, which was described in the appendix of [12] and reviewed in section 3. The orientifold acts as a reflection on the quiver diagram generalizing figure 2 to arbitrary n. The groups $\mathrm{SU}(N_1)$ and $\mathrm{SU}(N_{n+1})$ are identified with themselves with a symplectic projection (this is only consistent when N_1 and N_{n+1} are both even), while the group $\mathrm{SU}(N_2)$ is identified (up to an outer automorphism exchanging fundamentals and anti-fundamentals, due to the orientation reversal) with $\mathrm{SU}(N_{2n})$, $\mathrm{SU}(N_3)$ with $\mathrm{SU}(N_{2n-1})$, and so on (up to $\mathrm{SU}(N_n)$ which is identified with $\mathrm{SU}(N_{n+2})$). All in all, we end up with a gauge group

$$USp(N_1) \times \mathrm{SU}(N_2) \times \mathrm{SU}(N_3) \times \cdots \times \mathrm{SU}(N_n) \times USp(N_{n+1}). \tag{4.6}$$

The matter content still includes bi-fundamentals Q and anti-bi-fundamentals $\widetilde{Q}$ between all adjacent group factors in (4.6). The superpotential is similar to (4.1), except that on the two edges we have additional quartic terms coming from the projection of (4.1) using the identification described above:

$$W = Q^{(1)}Q^{(1)}\widetilde{Q}^{(1)}\widetilde{Q}^{(1)} + \sum_{i=1}^{n}(-1)^i Q^{(i)}Q^{(i+1)}\widetilde{Q}^{(i+1)}\widetilde{Q}^{(i)} + (-1)^{n+1}Q^{(n+1)}Q^{(n+1)}\widetilde{Q}^{(n+1)}\widetilde{Q}^{(n+1)}, \tag{4.7}$$

where in the first and last terms the two Q's (and the two $\widetilde{Q}$'s) are contracted in the USp group, so that they give an anti-symmetric tensor of the adjacent SU group.

The cascade in this theory is very similar to the previous one, except for the steps involving the "edge nodes". Again, let us assume for simplicity that

$$\Lambda^{(1)} \gg \Lambda^{(3)} \gg \cdots \gg \Lambda^{(2)} \gg \Lambda^{(4)} \gg \cdots. \tag{4.8}$$

In the first step of the cascade we then need to perform a Seiberg duality on the $USp(N_1)$ group which has $N_f = 2N_2$ flavors. This proceeds as described in [40], and turns the gauge group into $USp(2N_2 - N_1 - 4)$. The field content involves new bi-fundamental and anti-bi-fundamental quarks $q^{(1)}$ and $\widetilde{q}^{(1)}$, and mesons $M^{(2,2)}$ in the (adjoint+singlet) representation of $\mathrm{SU}(N_2)$, M^A in the anti-symmetric tensor representation of $\mathrm{SU}(N_2)$ and $M^{\overline{A}}$ in the conjugate anti-symmetric representation. The superpotential, including the terms coming from the duality transformation as well as the relevant terms of (4.7), is

$$W = M^{(2,2)}q^{(1)}\widetilde{q}^{(1)} + M^A\widetilde{q}^{(1)}\widetilde{q}^{(1)} + M^{\overline{A}}q^{(1)}q^{(1)} + M^A M^{\overline{A}} - M^{(2,2)}Q^{(2)}\widetilde{Q}^{(2)}. \tag{4.9}$$

The fields M^A and $M^{\overline{A}}$ are massive and can be integrated out, leading to a quartic term $-q^{(1)}q^{(1)}\widetilde{q}^{(1)}\widetilde{q}^{(1)}$. The quartic terms involving the second node have been replaced by having an adjoint+singlet field $M^{(2,2)}$ with trilinear couplings, just as in the discussion of the previous subsection.

If we now dualize the group $\mathrm{SU}(N_3)$, then again this will give a mass to $M^{(2,2)}$ (and regenerate the quartic couplings involving this mode), and generate a new adjoint field $M^{(4,4)}$ with trilinear couplings. Continuing along the cascade, if n is even we finish the odd steps by dualizing $USp(N_{n+1})$, while if n is odd we finish by dualizing $\mathrm{SU}(N_n)$. In both cases, after this we return to a theory of the same form (up to some signs and conjugations) as the original theory. The total rank is reduced by

$$N_{\text{cascade}} = N_1 + 2 - 2N_2 + 2N_3 - \cdots - 2N_n + N_{n+1} + 2 \tag{4.10}$$

when n is even, and by

$$N_{\text{cascade}} = N_1 + 2 - 2N_2 + 2N_3 - \cdots + 2N_n - N_{n+1} \tag{4.11}$$

when n is odd.

As before, we can now perform a similar cascade involving the even nodes. The duality of $\mathrm{SU}(N_2)$ does not give an adjoint for the adjacent USp group (since the superpotential makes this massive), but it does give an adjoint for the adjacent SU group. As we go down the cascade, eventually all the adjoints become massive, and we go back to a theory with the same form as the original theory, just as in the previous case. We can then continue cascading until the ranks become too small to perform further Seiberg dualities, and a different analysis is required in the IR.

Finally, let us describe the global symmetries in this case. We now have $(n+1)$ vector-like $\mathrm{U}(1)$ global symmetries. There is still a classical $\mathrm{U}(1)_R$ symmetry (with all bi-fundamentals having charge 1/2), but there is no longer any axial $\mathrm{U}(1)$ consistent with (4.7). For generic values of the N_i (for which $N_{\text{cascade}} \neq 0$, as required in order to have a cascade) the $\mathrm{U}(1)_R$ symmetry is anomalous. As before, this $\mathrm{U}(1)_R$ is not invariant under the cascade, unless we rescale the bi-fundamentals by appropriate powers of Λ's.

5. The IR physics of the cascade: non-perturbative mass generation

We will now discuss the IR physics of the cascade in two special cases. We assume that the cascade proceeds as in the previous section, until we need to perform a duality on the $USp(N_1)$ node but its rank is not large enough.

The first case we discuss is the case discussed in section 3, in which we want to end up with $USp(0) \times \mathrm{SU}(1) \times \cdots$. For this we need to start higher up in the cascade with $N_1 = 2N_2 - 4$, and with $N_4 = N_3 + 1$. The $USp(N_1)$ theory now has $2N_2 = N_1 + 4$ flavors. In this case the low-energy dynamics of this theory does not involve any modification of the classical moduli space for the mesons $M^{(2,2)}$, M^A and $M^{\overline{A}}$. Rather, there is an effective superpotential implementing the classical constraints. If we denote the full anti-symmetric meson matrix of the $USp(N_1)$ theory by $\mathcal{M}$ ($\mathcal{M}_{ij} = M^A_{ij}$, $\mathcal{M}_{i(j+N_2)} = M^{(2,2)}_{ij}$, $\mathcal{M}_{(i+N_2)(j+N_2)} = M^{\overline{A}}_{ij}$ for all $i, j = 1, \cdots, N_2$), then the low-energy superpotential is [40] of the form (we will ignore constants and powers of Λ's in all our expressions here, as before)

$$W = \mathrm{Pf}(\mathcal{M}). \tag{5.1}$$

The other couplings of the mesons are the same as the last two terms in (4.9) above, which come from the quartic superpotential. Since the fields M^A, $M^{\overline{A}}$ are massive we can integrate them out, and end up with a superpotential

$$W = \det(M^{(2,2)}) - M^{(2,2)} Q^{(2)} \widetilde{Q}^{(2)}. \tag{5.2}$$

As described above, the next stage in the cascade involves dualizing the $\mathrm{SU}(N_3)$ group to a $\mathrm{SU}(\widehat{N}_3) = \mathrm{SU}(N_4 + N_2 - N_3) = \mathrm{SU}(N_2 + 1)$ group. In this step we replace $Q^{(2)}\widetilde{Q}^{(2)}$ by a new meson field $\widetilde{M}^{(2,2)}$, and the relevant terms in the superpotential become

$$W = \det(M^{(2,2)}) - M^{(2,2)}\widetilde{M}^{(2,2)} + \widetilde{M}^{(2,2)} q^{(2)}\widetilde{q}^{(2)}. \tag{5.3}$$

The fields $M^{(2,2)}$ and $\widetilde{M}^{(2,2)}$ are now massive; integrating them out means that we can replace $M^{(2,2)}$ in (5.3) by $q^{(2)}\widetilde{q}^{(2)}$, where the $\mathrm{SU}(\widehat{N}_3)$ indices are contracted and the $\mathrm{SU}(N_2)$ indices are not; we will denote this matrix by $(q\widetilde{q})^{(2,2)}$.

We can now continue down the cascade, dualizing all the odd nodes. Next, we need to analyze the low-energy dynamics of the $\mathrm{SU}(N_2)$ node. This node has $N_f = \widehat{N}_3 = N_2 + 1$ coming from the $q^{(2)}$ and $\widetilde{q}^{(2)}$ flavors, so its low-energy description is in terms of mesons and baryons, with an effective superpotential imposing the classical constraints relating the mesons and the baryons [41]. The mesons $\widehat{M}^{(3,3)}$ are an adjoint+singlet of $\mathrm{SU}(\widehat{N}_3)$, while the baryons $B^{(2)}$ and anti-baryons $\widetilde{B}^{(2)}$ are in the fundamental and anti-fundamental representations. Like any other gauge-invariant chiral operator, we can write $\det((q\widetilde{q})^{(2,2)})$ in terms of these mesons and baryons. In fact, there are several ways to do this. On one hand it is equal to the subdeterminant of the mesons, which is a polynomial of rank N_2 in the traces of the $(\widehat{N}_3 \times \widehat{N}_3)$ meson matrix $\widehat{M}^{(3,3)}$, of the form

$$\begin{aligned} \mathrm{subdet}(M) &= N_2! \sum_{l=1}^{\infty} \sum_{n_i=1; i=1,\cdots,l}^{N_2} \frac{(-1)^{l+N_2}}{l!} \delta_{\sum_i n_i, N_2} \prod_{i=1}^{l} \frac{\mathrm{tr}(M^{n_i})}{n_i} \\ &= \mathrm{tr}(M)^{N_2} + \cdots + (N_2 - 1)!(-1)^{1+N_2} \mathrm{tr}(M^{N_2}). \end{aligned} \tag{5.4}$$

On the other hand, it is equal to a product of baryons $B^{(2)}\widetilde{B}^{(2)}$, with a contraction of their $\mathrm{SU}(\widehat{N}_3)$ indices. These two expressions are the same classically, and in this theory this equivalence remains true also quantum-mechanically; the superpotential which imposes that the moduli space is equal to the classical moduli space makes these two expressions the same in the chiral ring (namely, they are the same up to the addition of non-chiral operators).

At low energies the $\mathrm{SU}(N_2)$ group is gone. Making a specific choice of writing the determinant operator $\det((q\widetilde{q})^{(2,2)})$ using baryons rather than mesons (all other choices are equivalent in the superpotential), we obtain an effective action

$$W = \widehat{M}^{(3,3)}(B^{(2)}\widetilde{B}^{(2)} - q^{(3)}\widetilde{q}^{(3)}) - \det(\widehat{M}^{(3,3)}) + B^{(2)}\widetilde{B}^{(2)}. \tag{5.5}$$

Next, we want to perform a duality on the $\mathrm{SU}(N_4)$ theory. This turns $q^{(3)}\widetilde{q}^{(3)}$ into a new meson $M^{(3,3)}$, that couples trilinearly to new quarks $B^{(3)}$ and $\widetilde{B}^{(3)}$. The equation of motion of $M^{(3,3)}$ now relates $\widehat{M}^{(3,3)}$ to bilinears of these quarks. Integrating out all the massive fields reduces the superpotential to

$$W = B^{(2)}B^{(3)}\widetilde{B}^{(3)}\widetilde{B}^{(2)} - \det(B^{(3)}\widetilde{B}^{(3)}) + B^{(2)}\widetilde{B}^{(2)}, \tag{5.6}$$

where in the second term the quarks are contracted to give an adjoint+singlet of $\mathrm{SU}(\widehat{N}_3)$.

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The quiver theory that we ended up with is the same as the one described in figure 6 (identifying $\widehat{N}_3 = N_c$). The superpotential we find is also the same as we found there (up to the irrelevant determinant operator in (5.6)). In the derivation from the cascade it is not obvious that the mass term in (5.6) is related to a one-instanton effect in the "$USp(0)$" gauge group that we get at the end. However, it is easy to verify that such a one-instanton term would have the same quantum numbers (=the same anomalous R-charge) as the term we got.[2] Note that a one-instanton term in a specific gauge group in a cascading theory does not translate into a one-instanton term higher up in the cascade. This is evident from the computation in this section, in which the stringy instanton effect arises not from instantons but from strongly coupled dynamics of the gauge groups higher up in the cascade. Even though a single Seiberg duality transforms a one-instanton term into a one-instanton term, this is not true of the full cascade, which involves multiple Seiberg dualities on all the nodes (including nodes which are "flavor" nodes from the point of view of the instanton).

Similarly, we can analyze the case of $N_1 = 2N_2 - 2$, which formally ends up after the cascade step with $\widehat{N}_1 = -2$. In this case, the low-energy dynamics of the $USp(N_1)$ theory leads to a quantum modified moduli space [40]. So, instead of the terms involving $\det(M^{(2,2)})$ in the superpotential, we get constraint terms $\lambda(\det(M^{(2,2)}) - 1)$ with a Lagrange multiplier λ. All the later dualities are not modified, so at the end of the cascade step in this case we obtain the same superpotential as (5.6), but with the last term replaced by $\lambda(B^{(2)}\widetilde{B}^{(2)} - 1)$.

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References

[1] R. Blumenhagen, M. Cvetič and T. Weigand, *Spacetime instanton corrections in 4D string vacua — the seesaw mechanism for D-brane models*, *Nucl. Phys.* **B 771** (2007) 113 [hep-th/0609191].

[2] L.E. Ibáñez and A.M. Uranga, *Neutrino majorana masses from string theory instanton effects*, *JHEP* **03** (2007) 052 [hep-th/0609213].

[2] This would not be the case for the cascade involving only SU groups that we discussed in section 4.1, consistent with the fact that no mass is generated by the stringy instanton in that case.

[3] B. Florea, S. Kachru, J. McGreevy and N. Saulina, *Stringy instantons and quiver gauge theories*, *JHEP* **05** (2007) 024 [hep-th/0610003].

[4] M. Haack, D. Krefl, D. Lüst, A. Van Proeyen and M. Zagermann, *Gaugino condensates and D-terms from D7-branes*, *JHEP* **01** (2007) 078 [hep-th/0609211].

[5] N. Akerblom, R. Blumenhagen, D. Lüst, E. Plauschinn and M. Schmidt-Sommerfeld, *Non-perturbative SQCD superpotentials from string instantons*, *JHEP* **04** (2007) 076 [hep-th/0612132].

[6] O.J. Ganor, *A note on zeroes of superpotentials in F-theory*, *Nucl. Phys.* **B 499** (1997) 55 [hep-th/9612077].

[7] E. Witten, *Non-perturbative superpotentials in string theory*, *Nucl. Phys.* **B 474** (1996) 343 [hep-th/9604030].

[8] S.A. Abel and M.D. Goodsell, *Realistic Yukawa couplings through instantons in intersecting brane worlds*, hep-th/0612110.

[9] D.-E. Diaconescu, R. Donagi and B. Florea, *Metastable quivers in string compactifications*, *Nucl. Phys.* **B 774** (2007) 102 [hep-th/0701104].

[10] M. Bianchi and E. Kiritsis, *Non-perturbative and flux superpotentials for type-I strings on the Z_3 orbifold*, hep-th/0702015.

[11] M. Cvetič, R. Richter and T. Weigand, *Computation of D-brane instanton induced superpotential couplings-Majorana masses from string theory*, hep-th/0703028.

[12] R. Argurio, M. Bertolini, S. Franco and S. Kachru, *Metastable vacua and D-branes at the conifold*, *JHEP* **06** (2007) 017 [hep-th/0703236].

[13] R. Argurio, M. Bertolini, G. Ferretti, A. Lerda and C. Petersson, *Stringy instantons at orbifold singularities*, arXiv:0704.0262.

[14] M. Bianchi, F. Fucito and J.F. Morales, *D-brane Instantons on the T^6/Z_3 orientifold*, arXiv:0704.0784.

[15] L.E. Ibanez, A.N. Schellekens and A.M. Uranga, *Instanton induced neutrino Majorana masses in CFT orientifolds with MSSM-like spectra*, arXiv:0704.1079.

[16] N. Akerblom, R. Blumenhagen, D. Lust and M. Schmidt-Sommerfeld, *Instantons and holomorphic couplings in intersecting D-brane models*, arXiv:0705.2366.

[17] S. Antusch, L.E. Ibanez and T. Macri, *Neutrino masses and mixings from string theory instantons*, arXiv:0706.2132.

[18] S. Franco, A. Hanany, D. Krefl, J. Park, A.M. Uranga and D. Vegh, *Dimers and orientifolds*, arXiv:0707.0298.

[19] R. Blumenhagen, M. Cvetic, D. Lust, R. Richter and T. Weigand, *Non-perturbative Yukawa couplings from string instantons*, arXiv:0707.1871.

[20] M. Billo, M. Frau and A. Lerda, *$N = 2$ instanton calculus in closed string background*, arXiv:0707.2298.

[21] M.R. Douglas and G.W. Moore, *D-branes, quivers and ALE instantons*, hep-th/9603167.

[22] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB-resolution of naked singularities*, *JHEP* **08** (2000) 052 [hep-th/0007191].

[23] P.S. Aspinwall and S.H. Katz, *Computation of superpotentials for D-branes*, *Commun. Math. Phys.* **264** (2006) 227 [hep-th/0412209].

[24] O. Aharony, A. Buchel and A. Yarom, *Holographic renormalization of cascading gauge theories*, *Phys. Rev.* **D 72** (2005) 066003 [hep-th/0506002].

[25] R. Argurio, M. Bertolini, S. Franco and S. Kachru, *Gauge/gravity duality and meta-stable dynamical supersymmetry breaking*, *JHEP* **01** (2007) 083 [hep-th/0610212].

[26] S. Kachru and E. Silverstein, *4D conformal theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

[27] A.E. Lawrence, N. Nekrasov and C. Vafa, *On conformal field theories in four dimensions*, *Nucl. Phys.* **B 533** (1998) 199 [hep-th/9803015].

[28] I.R. Klebanov and E. Witten, *Superconformal field theory on threebranes at a Calabi-Yau singularity*, *Nucl. Phys.* **B 536** (1998) 199 [hep-th/9807080].

[29] A.M. Uranga, *Brane configurations for branes at conifolds*, *JHEP* **01** (1999) 022 [hep-th/9811004].

[30] K. Intriligator, N. Seiberg and D. Shih, *Dynamical SUSY breaking in meta-stable vacua*, *JHEP* **04** (2006) 021 [hep-th/0602239].

[31] R. Kitano, H. Ooguri and Y. Ookouchi, *Direct mediation of meta-stable supersymmetry breaking*, *Phys. Rev.* **D 75** (2007) 045022 [hep-ph/0612139].

[32] L. Görlich, S. Kachru, P.K. Tripathy and S.P. Trivedi, *Gaugino condensation and nonperturbative superpotentials in flux compactifications*, *JHEP* **12** (2004) 074 [hep-th/0407130].

[33] P.K. Tripathy and S.P. Trivedi, *D3 brane action and fermion zero modes in presence of background flux*, *JHEP* **06** (2005) 066 [hep-th/0503072].

[34] N. Saulina, *Topological constraints on stabilized flux vacua*, *Nucl. Phys.* **B 720** (2005) 203 [hep-th/0503125].

[35] R. Kallosh, A.-K. Kashani-Poor and A. Tomasiello, *Counting fermionic zero modes on M5 with fluxes*, *JHEP* **06** (2005) 069 [hep-th/0503138].

[36] E. Bergshoeff, R. Kallosh, A.-K. Kashani-Poor, D. Sorokin and A. Tomasiello, *An index for the Dirac operator on D3 branes with background fluxes*, *JHEP* **10** (2005) 102 [hep-th/0507069].

[37] D. Lüst, S. Reffert, W. Schulgin and P.K. Tripathy, *Fermion zero modes in the presence of fluxes and a non-perturbative superpotential*, *JHEP* **08** (2006) 071 [hep-th/0509082].

[38] N. Seiberg, *Electric-magnetic duality in supersymmetric nonabelian gauge theories*, *Nucl. Phys.* **B 435** (1995) 129 [hep-th/9411149].

[39] M.J. Strassler, *The duality cascade*, hep-th/0505153.

[40] K.A. Intriligator and P. Pouliot, *Exact superpotentials, quantum vacua and duality in supersymmetric* $\mathrm{Sp}(N_c)$ *gauge theories*, *Phys. Lett.* **B 353** (1995) 471 [hep-th/9505006].

[41] N. Seiberg, *Exact results on the space of vacua of four-dimensional SUSY gauge theories*, *Phys. Rev.* **D 49** (1994) 6857 [hep-th/9402044].